Chapter 9

COURSE I STATICS

9.1 Structural mechanics course outlines

The basic idea for the suite of courses is that structural mechanics may be conveniently separated into two interrelated parts,

1. Computational structural mechanics and analysis

2. Structural analysis and design

A clear understanding of the fundamentals of the first topic allows the easy transition to the more difficult subject of structural design. It is anticipated that undergraduate Computational Structural Mechanics will be divided into five subjects, levels (I to V), so that by the senior years of the course the student will appreciate modern computer methods and be prepared to move to the use of commercial software with a broad range of applications. In these five courses it is expected that the following topics will be covered.

I Determinate structures of the types, trusses, beams, frames and grids; force determination. Bending of beams, differential equations of equilibrium of a beam, distributed loading.

II Bending theory, symmetric, unsymmetric sections. Principal axes. Axial and shear stresses in solid and thin walled sections. Stress transformations and principal stresses. Definition of strains, and stress-strain relationships, Hooke's law, and constitutive matrix. Contragredient principle, deflection calculations.

III Member flexibility and stiffness matrices.Indeterminate structures; trusses, beams, frames and grids. Force and stiffness methods of analysis. Torsion of closed thin walled sections. Loads and support displacements. Direct stiffness methods

IV More on defelctions. Buckling theory and calculation of structure critical loads; beams and frames.

V Vibration theory and the determination of the natural frequencies and modes of vibration. Trusses, beams, frames and grids. Dynamic response, earthquake accelerations.

In the study programme presented in Chapter 9 for the first course the contents will cover determinate trusses, beams, frames and grids. Included is a study of reaction calculations of rigid bodies supported in a statically determinate manner. All course material is available on a CDROM. It contains.

1. Lecture notes of the complete theory.
2. Lecture summaries for each course.
3. A set of tutorials for each course.
4. The data file (DATN.DAT) with the necessary material for student assignments.
5. An appropriate subset of the STATICS-2020 software.
6. Explanatory slides of the structures studied in the course.
7. Email help address available to students for lecture and tutorial help.

9.2 Computational structural analysis-Statics

9.2.1 Lecture 9.1 Matrix-Vector Transformations

1.1 Introduction to elementary matrix algebra

Tutorial 1 introduces simple matrix operations. Students can execute STATICS-2020 and under the HELP menu check the matrix commands that are available. Make a list of hese commands. Input data from the dialog window may be used to explore command use.

Basic Theory-Vectors

Explain the meaning of a vector and the means for its representation. **Force:** magnitude $\bar{F}$ direction θ, and as a vector $\vec{F}$ or $\mathbf{F}$.
Displacement: magnitude $\bar{r}$, direction θ, as vector $\vec{r}$ or $\mathbf{r}$.
Position: magnitude $\bar{X}$, direction θ, as vector $\vec{X}$ or $\mathbf{X}$.
Components: $\{F\} = \left\{ \begin{array}{c} F_x \\ F_y \end{array} \right\}$; $\{r\} = \left\{ \begin{array}{c} r_x \\ r_y \end{array} \right\}$; $\{X\} = \left\{ \begin{array}{c} X_x \\ X_y \end{array} \right\}$
Magnitude:

$$\begin{aligned} \bar{F} &= \sqrt{F_x^2 + F_y^2} &&= (\mathbf{F}^T\mathbf{F})^{1/2} \\ \bar{r} &= \sqrt{r_x^2 + r_y^2} &&= (\mathbf{r}^T\mathbf{r})^{1/2} \\ \bar{X} &= \sqrt{X_x^2 + X_y^2} &&= (\mathbf{X}^T\mathbf{X})^{1/2} \end{aligned}$$

1.2 Transformation of components of vectors for rotation of axes

Read the section 1.2.2 of the text, with the discussion of coordinate axes. See Figure 1.1 and derive the orthogonal transformations.

$$\begin{aligned} \mathbf{F} &= \mathbf{L}^T\mathbf{F}'\,; & \mathbf{F}' &= \mathbf{LF} \\ \mathbf{r} &= \mathbf{L}^T\mathbf{r}'\,; & \mathbf{r}' &= \mathbf{Lr} \\ \mathbf{X} &= \mathbf{L}^T\mathbf{X}'\,; & \mathbf{X}' &= \mathbf{LX} \end{aligned}$$

1.4 Change of axes origin

Read section 1.2.3 with reference to Figure 1.2. Explain how the section 1.2.2. is concerned with a force acting at a point and the change in its components for a rotation of axes. In section 1.2.3 the interest is in the transfer of force and moment components from a point 'P' to its statically equivalent components at the origin 'O'. Understand the meaning of statically equivalence as this is an important concept. Derive the equation (1.6) and show that the moment component M in equation (1.6) can be obtained from the Z component of the cross product $\vec{M} = \vec{r} \times \vec{F}$ or alternately derived by using one component of the force at a time. Note that for a truss member forces at a node are statically equivalent to the applied forces at that node.

1.5 Change of displacement components from origin

Now consider the situation of displacements and infinitesimal rotations at the origin O. By using one component at a time verify equations (1.8) and (1.9). Hence using the three dimensional case show that the displacements at P$(x, y, z)$for the displacement θ_0 are given by the cross product $\vec{r}_P = \vec{\theta}_0 \times \vec{X}_P$.

9.2.2 Tutorial 1

T1.1 Plot the following vectors and determine their magnitudes and directions in each case.

$$1.\{F\} = \left\{ \begin{array}{c} 10 \\ 15 \end{array} \right\}; \left\{ \begin{array}{c} 15 \\ -10 \end{array} \right\}; \left\{ \begin{array}{c} -10 \\ 15 \end{array} \right\}; \left\{ \begin{array}{c} -10 \\ -15 \end{array} \right\} kN \qquad \text{force vectors}$$

$$2.\{r\} = \left\{ \begin{array}{c} 20 \\ 30 \end{array} \right\}; \left\{ \begin{array}{c} 20 \\ -20 \end{array} \right\}; \left\{ \begin{array}{c} -20 \\ 30 \end{array} \right\}; \left\{ \begin{array}{c} -20 \\ -30 \end{array} \right\} mm \text{ displacement vectors}$$

$$3.\{X\} = \left\{ \begin{array}{c} 2 \\ 3 \end{array} \right\}; \left\{ \begin{array}{c} 3 \\ -2 \end{array} \right\}; \left\{ \begin{array}{c} -2 \\ 3 \end{array} \right\}; \left\{ \begin{array}{c} -2 \\ -3 \end{array} \right\} m \qquad \text{position vectors}$$

T1.2 Execute STATICS-2020 and familiarize the use of the DATN.DAT input file, opening Exercise, Readme and Help menus. Open the main dialog input and examine the various command options for matrix operations in the drop down menus on the left hand side of the input dialog window. Undertake the assignments in 2-5 of **T1.3** using these commands. Print DATN.OUT and submit results.

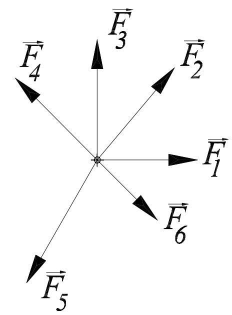

Figure 9.1: Forces acting at a point

T1.3 Open the INPUT dialog and using the MATRIX commands, undertake the following exercises. In each case print the results and print at the end of the session.

1. C=A+B
   $$A = \begin{bmatrix} 3 & 2 & 5 \\ 1 & 4 & 7 \end{bmatrix}; \qquad B = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \end{bmatrix}$$
2. C=AB
   $$A = \begin{bmatrix} 3 & 2 & 5 \\ 1 & 4 & 7 \end{bmatrix}; \qquad B = \begin{bmatrix} 1 & 2 \\ 1 & 2 \\ 1 & 2 \end{bmatrix}$$
3. C=A^{T}A
   where A is the column vector, A=[3 3 3]^T Use the command SQREL C to obtain its magnitude and check the result.
4. If F is the matrix of the vector components of the force $\vec{F}$
   $F = \begin{Bmatrix} 20 \\ 10 \end{Bmatrix}$ use $F' = LF$ to calculate the components F' for axes rotations of 45^o and 30^o respectively. In both cases show that the magnitude of F' has not changed from that of F.
5. The position vector of a point P in the X', Y' coordinates obtained by a rotation of 30^o has the values (20, 20) Plot the position of P, showing the coordinate axes. Calculate the components in the X, Y coordinates using equation (1.5)

9.2.3 Lecture 2 Equilibrium of forces

Read and explain sections 1.4 and 1.5. These sections are an introduction to the analysis of trusses, beams and frames. In order to obtain member forces and structure reactions the equilibrium of forces and moments acting at a point must be studied.

Equilibrium of forces at a point and of forces and moments acting on a planar rigid body

These two topics are essential for the calculation of forces in trusses and beams.

1. Discuss the conditions for the equilibrium of forces acting on both a point and a planar rigid body.

2. Equilibrium of forces acting at a point, see Figure 9.1.

   (a) vector form $\sum \vec{F_i} = 0$

   (b) component form $\sum F_i = \begin{bmatrix} F_{xi} \\ F_{yi} \end{bmatrix}$

   (c) Closure of the force polygon - graphical form. Consider the special cases of two forces and three forces acting at a point. The first must be collinear and equal and opposite for equilibrium the later must form a triangle.

3. Equilibrium of a planar rigid body.
   Forces systems $[F_i] = (F_{xi}\ F_{yi}\ M_i)^T$ act at points $P_i(x_i,\ y_i)$. Equilibrium is checked by finding the statically equivalent forces at a given point, for convenience the origin of the coordinate system. Read section 3.2 of the text for the theory, equations (3.1) to (3.5). Explain why the number of reactions in the plane must be equal to 3 for static determinacy and also for displacement stability.

9.2.4 Tutorial 2

Exercise on equilibrium calculations. See sections 1.2.1 to 1.2.3.

T2.1 The forces $\vec{F_1} = (2,\ 1)$ and $\vec{F_1} = (-3,\ 1)$ act at the point (0,0). The force $\vec{F_3}$ equilibrates the resultant of these two forces. Determine the magnitudes of the components of $\vec{F_3}$. Draw the triangle of forces ($\vec{F_1}\ +\ \vec{F_2}\ +\ \vec{F_3}$).

T2.2 The truss in Figure 1.3 has the angle $\beta = 45^o$. Use equation (2.3) to write the equilibrium of forces at node 3. Hence solve these equations to determine member forces (1) and (2).

T2.3 The force $\vec{F} = (10,\ 20\ 10) = (F_x,\ F_y,\ M)$ has these components applied at point P(7, 5) in the X, Y plane. Find the statically equivalent components at the origin. (use equation 1.6).

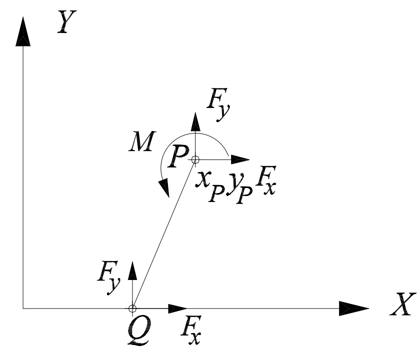

Figure 9.2: Equivalent forces on X axis at Q to force at point P

T2.4 A force and moment $\vec{F} = (F_x, F_y, M)$ acts at the point P(x_p, y_p). Show, using equation (1.6), that the statically equivalent force $(F_x, F_y, 0)$ applied on the $0X$ axis at the point Q has the X coordinate,
$X_Q = \frac{1}{F_y}(-y_P F_x + x_P F_y + M) = \frac{M_0}{F_y}$
Hence calculate X_Q for the force in **T2.3**.

9.2.5 Lecture 3 Truss structures

In this lecture the concept of structure is introduced and first read sections 1.4 and 1.5. Read Chapter 2, sections 2.2, 2.2.1 and 2.2.2. An idealized truss structure is composed of members that transmit axial forces only and nodes. Member forces are either axial tension or compression. Thus in Figure 9.3 the truss has three nodes and three members and three reaction forces. All structures require three data sets and these will be the matrices labelled A, B and C. A for coordinates, B for member end node numbers and C for reaction node numbers respectively. These values for the truss in Figure (9.3) are given below.

coordinates-A			**members-B**			**reactions-C**	
node	x-y values		member	i-j node numbers		node	angle
1	0	0	1	1	2	1	0
2	a	h	2	1	3	2	-90^o
3	L	0	3	2	3	3	0

See also Chapter 2, Figures 2.5 and 2.6. Execute STATICS-2020 and under the Exercise menu, view the trusses that have been preprogrammed into the software. For these trusses, the TRUSEX command can be used to invoke the data preparation of the A, B and C

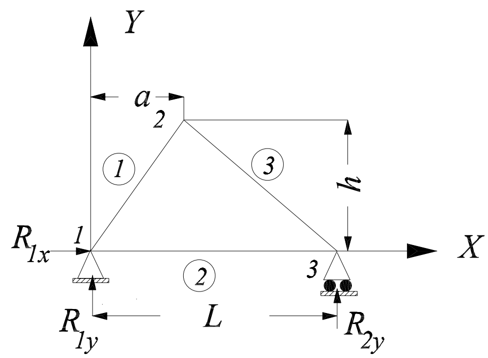

Figure 9.3: Three node-three bar truss

matrices. To start exercises on truss analysis a simple 3 node, 3 bar truss is given in the DATN.DAT file under the separator A0. For this truss the A, B and C matrices are read using the LOADR and LOADI commands and the data may be changed by editing the file, see the command separator A0 on the DATN.DAT file. The basic principles to be learnt are set out in the sections of Chapter 2. That is, every member is in equilibrium under the member forces acting at its extremities, see Figure 2.3. Member *node* forces are of opposite sign to member forces. Every node is in equilibrium under the action of:

(1.) Externally applied node forces.
(2.) Internal member node forces.
(3.) Reaction forces at the support nodes.
(4.) Every member is in equilibrium under the member forces acting at its extremities (nodes).

Study the conditions for determinacy, indeterminacy and stability of truss structures from equation (2.7). Examine Figures 2.4(a) and (b) and derive the expressions for global components of forces acting on the ends of an individual member. Explain equation (2.10) and the formation of equation (2.2) for all member forces and reactions. Read section 2.2.3. In DATN.DAT change the data so that after executing the TRUSS A B C D command a RETURN statement is inserted. This ends the command sequence that was started following A0. When SUBMIT A0 is now used the D matrix contains the 6 equations of joint equilibrium. Print this matrix and explain the significance of the terms in each of its columns.

9.2.6 Tutorial 3

This tutorial is for understanding the analysis of statically determinate trusses. All trusses are available on the DATN.DAT file, see examples A0 to A9 and A15.

T3.1 Run STATICS-2020 and enter EXERCISE window. Open the various truss files (T1,T2, ...) and view the examples that are available using the SUBMIT command. command. Open the command dialog window and input SUBMIT A0. This executes the commands on DATN.DAT for the analysis of the simple 3 bar-3node truss in Figure 9.3.(Remember that the RETURN statement inserted in Lecture 3 must first have been deleted). Choose dialog PLTRUS A B C N=1. This will view the truss geometry. Return to the main dialog and enter the command PLTRUS A B C S R N=2. This produces a view of the truss showing applied load and truss members now in different colours, blue for tension, red for compression and green for zero force. The command JOINEQ A B C R S N=2 has calculated X and Y components of all the forces acting on node 2 and summed the columns. PRINT S1 to view this result and prove that the sums of columns are equal to zero. Enter the command PLTJNT S1. This gives the vector plot of the results of the previous call to JOINEQ. The polygon (triangle) of forces at node 3 is shown to close, proving equilibrium of the node. Make a hard copy of the plot by printing the file 'screen.plt'. Also make a hard copy of the file DATN.OUT.

T3.2 Edit the DATN.DAT file, A0 data set changing the coordinate of node 2 to (10, 10). Repeat the exercise **T3.1**. Note the difference in the member forces. The last three rows of the member forces (4, 5 and 6) in this case are the reactions. Prove that the reactions are in equilibrium with the applied loads.

T3.3 Edit DATN.DAT, A0 data set changing the coordinate of node 3 to (10, 0) and then repeat the exercise **T3.1**. Tabulate the member forces and note the differences with previous results.

T3.4 Edit DATN.DAT and insert a RETURN command immediately after the TRUSS command. Now rerun the SUBMIT A0 command. Print the matrix D(6×6) and identify the columns of this matrix for member and reaction forces.

T3.5 Analyse truss (1) in Figure 2.8 using the SUBMIT A1 command. Plot the truss and print member forces. Identify tension and compression member forces and explain the flow of forces through the truss to the reactions. Use JOINEQ and PLTJNT commands for nodes 3 and 6. Run the command TOPOL A B B1 and PRINT B1, Explain the reason for the number 3 in all the diagonal terms and the off diagonal terms of 1. Check the equilibrium of the rigid body composed of members 7-8-9, nodes 4-5-6 using the appropriate member and node forces.

T3.6 Analyse truss (3) in Figure 2.9 using SUBMIT A3 command. Equate the force in member (3) to the value of the vertical reaction at node (1). Examine the truss in Figure 2.9 (4). How do you expect the force in member (3) for this truss to differ from that member (3) in the previous exercise? For the truss in Figure 2.9 (3) that you have analysed, take moments of forces, applied loads, reaction to left of node (8). Prove that the left-hand rigid body is in equilibrium. Why are the forces in members (1),(4),(24) and (25) all equal to zero? Are there load cases in which they are not equal to zero? If so, give some examples.

T3.7 Analyse the truss (6) in Figure 2.9 using SUBMIT A6. Examine the values of the forces in members (1) and (11) and explain their signs and magnitudes. Why is one of these forces tension and the other compression? Use PLTRUS A B C S R N=2 to plot the truss marking the magnitudes of the forces (tension, compression and zero).

T3.8 Analyse the truss in Figure 2.10 (A7). Run the TOPOL A B B1 command and examine the diagonal terms of S2. Explain the significance of diagonal terms. Plot the truss and mark the member forces.

T3.9 Analyse the truss (13) in Figure 2.13 with the SUBMIT A8 command. What is the advantage of having the top chord curved making the truss deeper towards the centre of the span? Plot the truss and print the member forces, putting force magnitudes on the members. Use the forces in members (9) to (12) to explain your conclusions. Take a vertical section through members (1), (9) and (18) and check the equilibrium of the vertical forces on the left of the section.

9.2.7 Lecture 4 Beam Free Body Diagrams

Free Body Diagram

This section discusses bending moments and shear forces in beams. Read sections 3.3.1 and 3.3.2 and revise Chapter 1 section 1.2.3 equation (1.6). Using the equation (1.6) and the principles of free body diagrams, the concepts of internal bending moment and shear forces are introduced. These internal forces are necessary to maintain the equilibrium of any portion of the beam. From the portion OX of the beam in Figure (9.4)b the following conditions apply.

$\sum F_y = 0$
Hence
$R_0 + F_1 + F_2 - V = 0$
So that
$V = R_0 + F_1 + F_2$

Also,
$\sum M_0 = 0$
$F_1 x_1 + F_2 x_2 + M - Vx = 0$
Hence substituting for V
$M = R_0 x + F_1(x - x_1) + F_2(x - x_2)$

For the infinitesimal length of beam shown in Figure (9.4) $\sum F_y = 0$ That is, $V - (V + dV) + wdx = 0$ (1) $w = \dfrac{dV}{dx}$	and $\sum M_B = 0$ neglecting 2nd order small quantities $-M - \dfrac{w\,dx^2}{2} + (M + d\,M) - V\,dx = 0$ (2) $V = \dfrac{d\,M}{d\,x}$; hence (3) $w = \dfrac{d^2 M}{d\,x^2}$

Now the concept is introduced of a node in a beam structure composed of beam elements between the nodes, see Figure 3.6. Derive the equations (3.10)and (3.13). Various

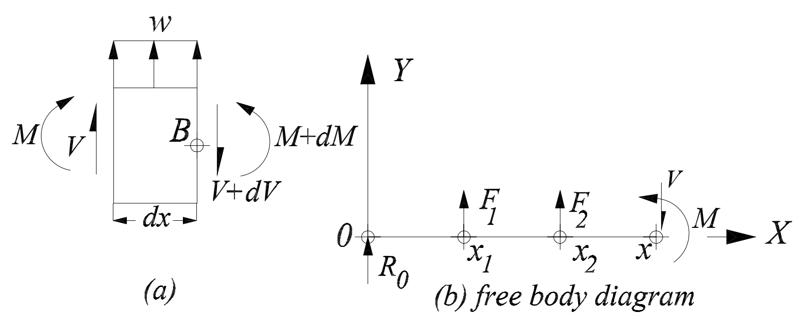

Figure 9.4: Infinitesimal beam element; expressions for shear and moment

simple beam types and their supports are shown in Figures 3.7 and 3.13. These types are classified,
(1.) Simply supported beam.
(2.) Beam with overhang ends.
(3.) Cantilever beam.
(4.) Beam with internal hinges.
Explain the meaning of each of these classifications.
Beam loads
In this study loads are classified as,

1. Concentrated on nodes, either a force or a moment.
2. Uniformly distributed force over the length of a member.
3. Consider the distributed load w per unit length on the length dx so that $F_y = wdx$. Then the equation $M_0 = xF_y$ may be integrated over the length L to show that a uniformly distributed load of w on the length L can be replaced by either,
   (a) wL at the centre of the length L.
   (b) $\frac{wl}{2}$ at each end of the length L.

For example prove that the moment of the replacement forces for distributed load about any point, e.g. the origin, is the same for both cases (a) and (b).

9.2.8 Tutorial 4

In Lectures 2 and 3 and Tutorials 2 and 3, the concept of equilibrium of forces acting at a point has been used to calculate member forces in statically determinate trusses. It

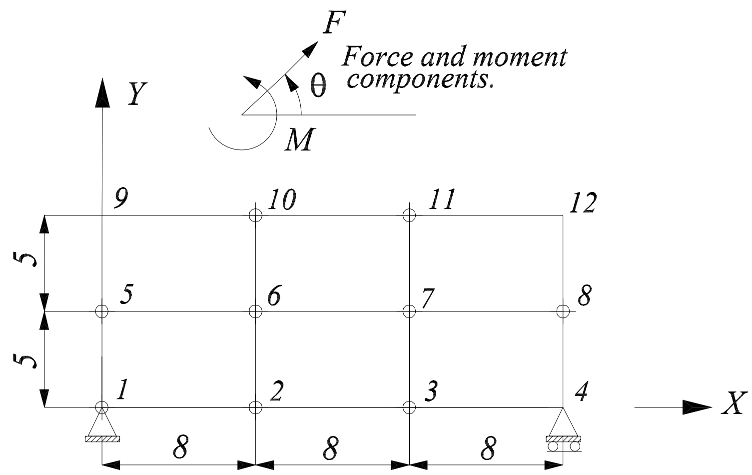

Figure 9.5: Rigid body reactions

will be recalled that the basis of this was the equation (2.3) for the transformation of force components for a rotation of the coordinate axes. In Tutorial 2, **T2.3** and **T2.4**, the additional concept of the transfer of forces and moment acting at one point P to a second point Q has been introduced. This is a development of the equation (1.6) in Chapter 1. The student must now read section 1.2.3. This transformation is basic to the understanding of the calculation of the reactions of rigid bodies and for the calculation of bending moments and shear forces in beams. Statically determinate beam analysis can be considered to be a repeated application of equation (1.6). In order to introduce beam analysis the concept of equilibrium of planar rigid bodies, supported in a statically determinate manner(3 reaction components in the plane), is first studied. Read the theory in sections 3.2.1 and 3.2.2 and the application to retaining walls in section 3.2.3. In this tutorial, there are three exercises on the DATN.DAT file. The first **R1** is a straight copy of the example in Figure 3.2 of the text, section 3.2.3. The second **R2** is an example of the analysis of a retaining wall assumed to be supported by two forces and a moment on its base. This is a more advanced application and uses the command RETWAL to set up the solution equations. It will not be discussed in this first course. The third example **R3** is based on the Figure (9.5) and is used to set up a variety of exercises. In Figure (9.5) a rigid body of dimensions (24×10), has 12 points on a uniform (8×5) grid. The three supports are at the nodes 1 and 4 as shown in the Figure (9.5). The basic data and commands for calculating reactions for this grid are given in Table 9.1. The first 12 rows of the input matrix A are setup to represent the points 1-12, the columns 3 and 4 in each row giving the coordinates of the point corresponding to that row, (e.g. row 5 is for point 5, etc.). The rows 13-15 are setup for the three reactions at the nodes 1 and 4. As setup A contains no active force or moment data. This is input using the MODIFY command from input dialog window and then SUBMIT R4. This process is illustrated in Table 9.1.

After this command, reactions may be viewed from the input dialog with PRINT R, and a hard copy produced from DATA.OUT. This should be made at the end of the run session as an intermediate hardcopy print will require the program to be restarted.

T4.1 Use SUBMIT R1 on the DATN.DAT file to run the data that is given in section 3.2.2. From the input dialog view the calculated reactions using PRINT R. Use statics to prove that these results equilibrate the applied loads.

T4.2 See Table 9.1. Use the SUBMIT command with R3 and R4 separators as discussed above to explore the application of forces and moments to the nodes of the rigid plate shown in Figure 9.5. For each example first use the SUBMIT R3 and then SUBMIT R4 command and print all results at the end of the session. Check each result by hand calculations.

Table 9.1

```
C in the data A matrix given below
C the loads must be modified to
C give meaningful results
R3
LOADR A R=15 C=5
0 0 0 0 0
0 0 8 0 0
0 0 16 0 0
0 0 24 0 0
0 0 0 5 0
0 0 8 5 0
0 0 16 5 0
0 0 24 5 0
0 0 0 10 0
0 0 8 10 0
0 0 16 10 0
0 0 24 10 0
1 0 0 0 90.0
1 0 0 0 0
1 0 24 0 90.0
RETURN
```

```
C use the MODIFY command from the
C INPUT DIALOG to change the data in A.
C Using R4 then produce a new set of
C reactions for this data.

R4
REACTN A B C
INVERT C T=2
MULT C B R
PRINT R
RETURN
```

Complete the following assignments (1) to (8).

Apply:

1. A vertical force of 100 units at node 2.
2. Vertical force of 100 units at node 7.
3. Moment of 100 units in turn at nodes 1,3,7,12.

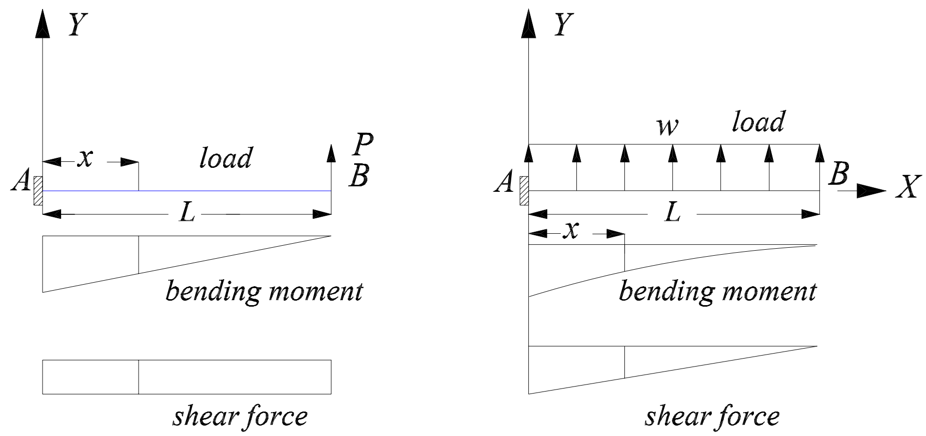

Figure 9.6: Cantilever beam concentrated and distributed load

4. Force of 200 units inclined at 30^o to the X axis at node 1.
5. Horizontal force of 100 units at node 5.
6. Change the support at node 4 to be roller on a vertical plane at node 9.
7. Repeat exercises (10 to (6) with the new support condition.

9.2.9 Lecture 5 Bending moment-shear force

Bending moment and shear force diagrams

Table 9.2 Cantilever beam

Concentrated load	Distributed load
$M_x = P(L - x)$	$M_x = \dfrac{w(L-x)^2}{2}$
$M_A = PL$	$M_A = \dfrac{wL^2}{2}$
$V_x = -P$	$V_x = -w(L - x)$
$\left\{ \begin{array}{c} \delta_B \\ \theta_B \end{array} \right\} = \dfrac{P}{6EI} \left\{ \begin{array}{c} 2L \\ 3 \end{array} \right\}$	$\left\{ \begin{array}{c} \delta_B \\ \theta_B \end{array} \right\} = \dfrac{wL^3}{24EI} \left\{ \begin{array}{c} 3L \\ 3 \end{array} \right\}$

Lecture 4 introduced the calculation of bending moments and shear forces in a statically determinate beam using the concept of free body diagrams. It was shown how a uniformly distributed load may be replaced in either of two ways described in section 9.9.2. In this lecture the four examples given in Figures 9.6 and 9.7 are worked in so far as the bending moment and shear force diagrams can be drawn (see theory given in section

9.9.1).

Case studies of beams

1. *Cantilever beam*

   (a) For the cantilever beam, load P is applied at the free end of the span L. The bending moment and shear force diagrams for P applied at a distance a from the left hand end $x < L$, are deduced from the case given in Table 9.2.

   (b) Distributed load
   The second cantilever case is for a distributed load w per unit length on the whole span L. Note the bending moment diagram varies quadratically and the shear force linearly along the beam length as shown in Figure 9.6. The case should be studied for which w per unit length is applied only over the portion between x_1 and x_2 of the beam, x_1 and x_2 being measured from the fixed end of the beam see Figure 9.8. Use the results in Figure 9.6 to obtain the result. Hint: In CD use the origin at C and replace x by $(x - x_1)$ and L by $(x - x_2)$. For AC replace $w(x - x_1)$ by equal forces at C and D.

Table 9.3 Simply supported beam

Concentrated load P at (a, b)	Distributed load w over whole span
$R_A = \frac{-Pb}{L}$; $R_B = \frac{-Pa}{L}$	$R_A = R_B = \frac{-wL}{2}$
$M_C = \frac{-Pab}{L}$	$M_C = \frac{-wl^2}{8}$
$M_x = \frac{-Pbx}{L} \quad x \leq a$	$M_x = \frac{-wx}{2}(L - x)$
$M_x = \frac{-Pa}{L}(L - x)\ x \geq a$	

2. *Simply supported beam.*

   (a) Concentrated load
   Two load cases are studied and bending moment and shear force diagrams drawn, see Figure 9.7. The first case is for a concentrated load P that is applied at the point that divides L into the lengths $(a,\ b)$, as shown in Figure 9.7. Note that the result for the point load given in Table 9.3 is easily applied for any values of $(a,\ b)$, and the maximum moment always occurs at the load point.

   (b) Distributed load
   The distributed load uniformly distributed over the whole span L, produces a quadratic variation in the bending moment, see Table 9.3. The case should be studied for which the distributed load w is applied over the length between x_1

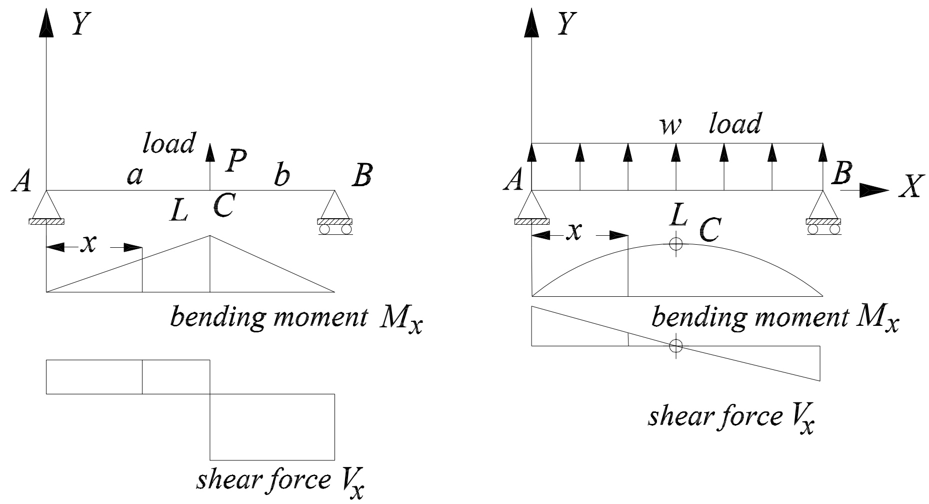

Figure 9.7: Simply supported beam; concentrated and distributed loads

and x_2 of the span, see Figure 9.8. If the bending moment diagram is drawn it can be proved that the moment at the centre of the loaded length , point E for w positive as shown in Figure 9.8, is given by,

$$M_E = \frac{1}{2}(M_C + M_D) - \frac{w(x_2 - x_1)^2}{8}$$

This result is true for any length CD over which a uniformly distributed load is applied. Now becasue the shear force varies linerly between C and D, it follows that the shear force at the point E mid way between C and D is given,

$$V_E = \frac{1}{2}(V_C + V_D)$$

9.2.10 Lecture 6 Beam discretization

Beam discretization

It was shown in Lecture 4 that beam analysis is essentially the solution of the differential equation,

$$M = \frac{d^2M}{dx^2}$$

If the distributed load is discretized at the nodes and $w = 0$ on elements then the bending moment M varies linearly between nodes, see Figure 3.6. Thus a beam is discretized by first dividing its length into a number of segments. See Figure 3.16 for beam examples and note the number of subdivisions. Also see the beam examples (18-30) on DATN.DAT and in particular (18-22) for statically determinate beams. The forces applied at a beam node are (F_Y, M_Z), see Figure 3.15. That is, transverse forces in the Y direction and a moment applied about the Z axis. The beam element forces, Figure 3.6 are derived in

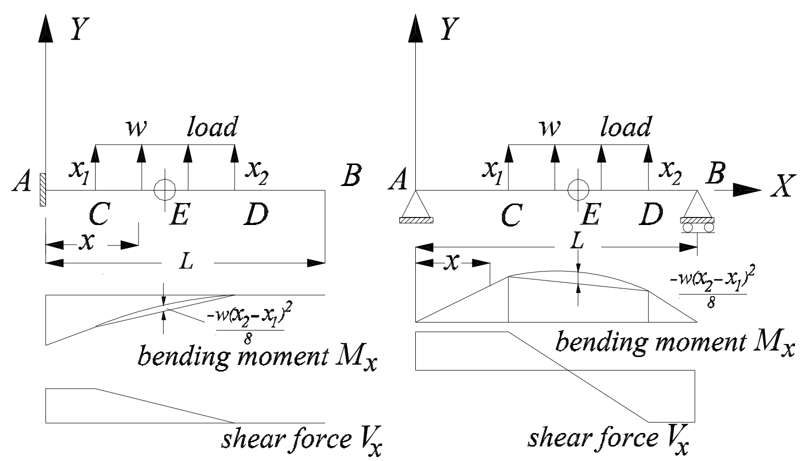

Figure 9.8: Partial distributed loading on cantilever and simply supported beams.

equations (3.12) and (3.13). (Read sections 3.3.1 and 3.3.2). STATICS-2020 has a group of commands available for beam analysis as described in section 3.3.3, and also read section 3.3.5. Once the beam has been idealized into members and nodes it can be analysed by the normal equations of node equilibrium. In this manner the member bending moments are obtained for the nodal forces. If uniformly distributed load has been applied to a member the procedure shown in Figure 9.8 is used to obtain the quadratic variation of bending moment within that member. Shear forces are obtained from element statics. Analysis of beams in this way may seem tedious. However the theory and the computer software are directly applicable to the more difficult problems of calculation of deflections, beam buckling and natural frequencies of vibration. It is also an introduction to frame and grid analyses and the extension to statically indeterminate beams is straight forward.

9.2.11 Tutorial 5

See section 3.7.1, exercises **B1** to **B6**. The exercises given in this Tutorial 5 are introductions to beam analysis and should be used as precursors to studying **B1** to **B6**.

T5.1 Read section 3.3.1 and 3.3.2. Draw the beams shown in Figure 3.8(a)-(c) and insert the reactions for each case. Explain how, in the beams where they occur, the hinges allow the calculation of all the reactions by statics alone. In all cases, the left hand support is a pin, reactions (R_x, R_y). If only vertical loads are applied, what is the magnitude of R_x at the node?

T5.2 Read section 3.7 and examine the beams shown in Figure 3.16. Classify the beams according to examples in Figures 3.7 and 3.8. Beams 3.16 (1), (3) and (6) are all simply supported spans. How do they differ?

T5.3 Prove the results in Lecture 5, Tables 9.2 and 9.3 for the bending moment and shear force diagrams for both cantilever and simply supported beams. From the results in Lecture 5 for partial loading calculate the bending moment and shear force diagrams for the load applied over the length CD from x_1 to x_2 on both cantilever and simply supported beams.

T5.4 A cantilever beam of span AB = L has an uniformly distributed load of 1 kN/metre applied over length from L/2 to L. Calculate the bending moment and shear force diagrams for L=10m.

T5.5 The exercise **B2** on the DATN.DAT file is for a cantilever beam of 10m span. Edit the DATN.DAT file and change the uniformly distributed load to be only on members 6-10. Run STATICS-2020 with the SUBMIT B1 command from the input dialog window. Plot the bending and shear force diagrams printing out their values. Show that the results are the same as were obtained in **T5.4**.

T5.6 The data for the problem **B5** on the DATN.DAT file is for the beam with an internal hinge shown in Figure 3.16 (5). The dimension in the Figure 3.16(b) is, L=16 units.

Run STATICS-2020 with the SUBMIT B5 command. Print E and F, applied loads, and hence ascertain the loads that were applied to the beam. Calculate and draw the bending and shear force diagrams. Print the values of moment (M) and shear (V). Verify these results by calculating the values using hand calculations. Also print and check the reactions (S).

9.2.12 Lectures 7−8 Determinate frames

Determinate frames

A simple example of a determinate frame type of structure is shown in Figure 4.1. The forces (10,20) on node 4 produce both axial and shear forces in members 1-2 and 2-4, as well as bending moments in all members. Consequently at the nodes, the three equilibrium equations,

$$\sum F_x = 0; \quad \sum F_y = 0; \quad \sum M_z = 0$$

must be satisfied. the student should now read Chapter 4, sections 4.1 and 4.2.1, 4.2.2. The basic member forces (F, M_i, M_j) acting on a frame member are shown in Figure (4.2a). A transverse shear (V) will, in general, be necessary to equilibrate the moments acting on the member ends $I - J$ and so the three member forces generate the member nodal forces (F'_x, F'_y, M_z) in the local, member coordinates axes, see Figure 4.2(b). Finally, these local forces are transformed into the (X, Y, Z) global coordinate axes components. As for the truss structure all nodal equations of equilibrium will be written together in the global coordinate system, see Figure 4.2(c). Examine the equation (4.1) that expresses the local coordinate components in terms of the basic member forces and explain how this equation is a combination of the equation (2.8) for a truss member and equation (3.13) for a beam member. Finally equation (4.2) embodies the transformation given in equation (1.5), for rotation of components from local to global axes. Note that no transformation is required for (M_z) which can simply be written as M because the moment is always about the Z axis. The equilibrium equations, 3 per node in terms of 3 basic forces per member and reaction components are written for all nodes in equation (4.7). For a simple determinate frame without internal member releases, for static stability there will be 3 reaction components. In STATICS-2020, the data for the frame analysis follows the same naming convention as for trusses and beams. That is, A stores nodal coordinates, B member topology and C reaction location and type information. The equilibrium equations are then generated (and if determinate, inverted), with the command

FRAMEQ A B C

A frame analysis command sequence for the frame in Figure 4.7(a), is given in problem 41(F9) in the DATN.DAT file. For this frame a load has been applied at node 6 in the X direction. The student should run STATICS-2020 and view Fram0.bmp for this frame and also Frame1.bmp for the indeterminate frame types that are also available in exercise (42-48).

Load generation

Read section 4.3 on node and member loads. Therein the available load types are discussed. These can be, concentrated nodal forces, (F_x, F_y, M) or distributed member loading (w per unit length of member), in either the positive local Y' or the global Y direction(see command option D=±1).

Examine the command sequence for (41)-F9 on DATN-DAT, and note that deflection calculations are carried out, the theory for which is beyond the scope of the present course on statics. The command to generate the data for the example (41) is,

FRAMEX E=1 D-?,?,?,?

See Chapter 4, for the options on the frame dimensions in D. Having read the load matrices E and F the nodal force vector LO is calculated with

FRAMLD B E F C=? D=?

9.2.13 Tutorial 6

See Figure 4.7(9) and also Fram0.bmp under Exercise main menu of STATICS-2020. In exercise (41) with separator F9 the frame dimensions are a=8.0, b=2.0, c=d=1.5 metres and a load of 100Kn is applied at node 6 in the negative Y direction. Node 1 is fully restrained against translation and rotation. Check that this is the data entered after separator F9 on the DATN.DAT file.

T6.1 Run STATICS-2020 and SUBMIT F9. Print the A, B and C matrices and check frame data generated by the command FRAMEX. Plot the bending moment and shear force diagrams. Make hard copies of these diagrams and annotate them from the print out of moments M, shear V and reactions S.

T6.2 Using the print outs of M,V,S and the applied loads, at node 2, draw the free body diagram of forces acting on the node and prove that they are in equilibrium.

T6.3 Prove that the reactions, S, equilibrate the applied load on node 6.

T6.4 For the frame nalysed in T6.1 consider the member 4 connecting nodes 2-5. Apply a distributed load of 10Kn/unit length on the member
(a) in the coordinate direction Y' of the member
(b) in global Y coordinate direction
In both cases calculate the nodal forces to be applied to nodes 2 and 5 in the global coordinate system using equations 4.8 and 4.2 where applicable. Edit the commands following F9 on the DATN.DAT file to include these loads in two separate analyses, and SUBMIT F9. Print the load vector LO in both cases and prove that the values there agree with the hand calculations.

T6.5 For the original load case (100 in the negative Y direction on node 6), use the PLTFRM command to draw the deflected shape produced by the analysis commands. Make a hard copy of this result and explain the deflected shape of the structure.

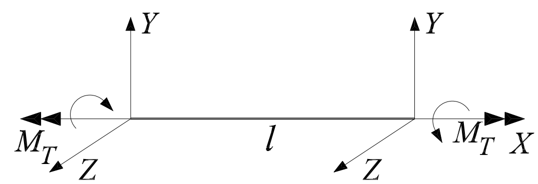

Figure 9.9: Positive torque or member twisting moment

9.2.14 Lectures 9-10 Determinate grids

Determinate or open grid structures

The basic theory of grid structures has been given in Chapter 5. Read Chapter 5, sections 5.1 and 5.2. It is found that in grid structures the concept of torsional moments not yet met in either truss, beam or plane frame analysis is encountered. The term open grid will apply to statically determinate grids as these structures having only 3 reaction components cannot have closed loops. Although the concept of torsional moments, and as a consequence twisting of members, is new it is found that the theory for grid analysis is remarkably similar to that for plane frame analysis and the transformations of member forces and nodal moments are readily developed by analogy. For the grid structure see Figure 9.9 three generalized forces act at a node. The grid is assumed to lie in the $X - Z$ plane. One of the nodal forces F_y is transverse to the plane of the grid, and the other two are moments (M_x , M_z) positive acting in clockwise sense when viewed from the origin (right hand screw rule). To distinguish between forces and moment vectors the moment vectors will be drawn with double headed arrows. The concept of twisting moment (or torque) M_T acting on the member nodes IJ is shown in its positive sense in Figure 9.9. The calculation of the internal shear stresses due to the torque M_T is simple only in the case of members of circular cross section.

Example of simple grid transformations

In Figure 9.10 a cantilever beam, nodes 1-2 supported at node 1 has a force and moments (F_y, M_x, M_z) of magnitudes (2,8,5) applied at node 2. Calculate the torque and bending moment in the members at nodes 2 and 1. In order to calculate the torque in the member use the inverse transformation of equation 5.2 to resolve (M_x, M_z) into (M'_x, M'_z) in the local member coordinates. From Figure 9.10, β positive as shown, $\cos\beta = 4/5$, $\sin\beta = 3/5$

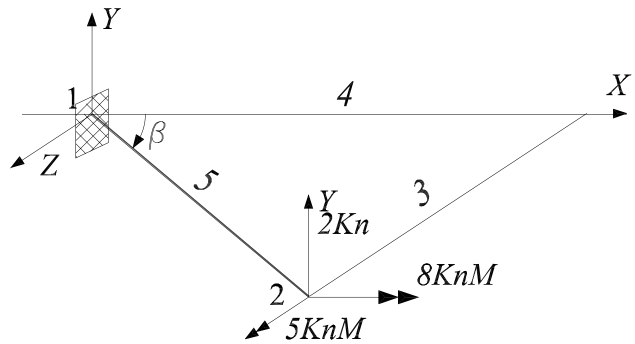

Figure 9.10: Cantilever subjected to bending and twisting moments

and the transformation of moments is,

$$\left\{ \begin{matrix} M'_x \\ M'_z \end{matrix} \right\} = \begin{bmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{bmatrix} \left\{ \begin{matrix} M_x \\ M_z \end{matrix} \right\}$$

That is at node 2,

$$\left\{ \begin{matrix} M'_x \\ M'_z \end{matrix} \right\} = \frac{1}{5} \begin{bmatrix} 4 & 3 \\ -3 & 4 \end{bmatrix} \left\{ \begin{matrix} 8 \\ 5 \end{matrix} \right\} = \frac{1}{5} \left\{ \begin{matrix} 47 \\ -4 \end{matrix} \right\}$$

giving the values of torque and bending moment,
$M_T = 47/5 \quad M = -(4/5)$
At node 1 the moments about the X and Z axes are given,

$$\left\{ \begin{matrix} M_x \\ M_z \end{matrix} \right\} = \left\{ \begin{matrix} 8 \\ 5 \end{matrix} \right\} + \left\{ \begin{matrix} -3 \\ 4 \end{matrix} \right\} 2 = \left\{ \begin{matrix} 2 \\ 13 \end{matrix} \right\}$$

Then in local member coordinates,

$$\left\{ \begin{matrix} M'_x \\ M'_z \end{matrix} \right\} = \frac{1}{5} \begin{bmatrix} 4 & 3 \\ -3 & 4 \end{bmatrix} \left\{ \begin{matrix} 2 \\ 13 \end{matrix} \right\} = \frac{1}{5} \left\{ \begin{matrix} 47 \\ 46 \end{matrix} \right\}$$

The reactions at node 1 will be equal and opposite to these moments and hence it is seen that at node 1,
$M_T = 47/5 \quad M = 46/5$
Note that the torque is the same as at node 1 and the bending moment has increased by the amount,
$(46 + 4)/5 = 10 = 2 \times 5$
From this example it is seen that for a grid, moment quantities $(M_x,\ M_z)$ form components of a moment vector $\vec{M}$, where as, in so far as grid action is concerned, F_y transforms from

one point to another as a scalar. Compare the plane frame for which in the plane forces (F_x, F_y) are components of a force vector $\vec{F}$ where as M_z, the out of plane moment transforms as a scalar. Thus for the grid structure there are three equilibrium equations per node,

$$\sum M_x = 0\,; \quad \sum M_z = 0\,; \quad \sum F_y = 0$$

For the individual member the transformation from member forces to global components is given in equations (5.1) to (5.5) and the combined equilibrium equations for all nodes, is given in equation (5.6), or (5.7). Examine the equation (5.1) and explain how this is composed of individual components, see equation (3.12). The data set for grid analysis follows the same naming convention as for trusses, beams and frames. That is, A stores nodal coordinates, B member topology and C reaction location and type. The command

GRIDEQ A B C

generates the equilibrium matrix EQ and if determinate inverts to produce the member force transformation matrix $[b]$. Two examples of determinate grid structures are provided in problems numbers 52(G1) and 54(G3) and are activated using the command,

GRIDEX E=? D=?,?,?,?

Where E=1 or 4 and the definitions of D are as given in Figures 5.7(1) and 5.8(4). The real purpose of grid analysis is for the design of more complicated bridge deck and floor beam systems as shown in Figures 5.9 and 5.10. These are indeterminate problems and are beyond the scope of the present course.

9.2.15 Tutorial 7

Tutorial 9.2.15 involves exercises in determinate grid analysis based on Figures 5.7 and 5.8. Run STATICS-2020 and under Exercises menu, view grid1.bmp and grid2.bmp. These are copies of the exercises involved in [G1] to [G3] in Chapter 5. Read data on DATN.DAT under separators G1 and G3 and understand the command sequences.

T7.1 The grid shown in Figure 5.7(1) is statically determinate and may be analysed using problem 52(G1), on DATN.DAT, using SUBMIT G1. apply the following loads, $M_X = 100,\ M_Z = 100,\ F_Y = 20$

1. Print moment M, shear force V, reactions S and torque T. From simple statics undertake the following exercises:
2. Check bending and twisting moments in members 1,2 and 3.
3. Prove reactions (S) equilibrate applied loading.
4. Draw free body diagrams of joints 2 and 3 and using output in M, V and X prove that the joints are in equilibrium.

T7.2 The determinate structure(light pole)is shown in Figure 5.8(4). For loads applied in the Y direction and moments M_x, M_z it behaves as a determinate grid. Notice the axes chosen. Plots will be always be in the $X - Y$ plane. The commands for analysis of this grid have been programmed on the DATN.DAT FILE under (54), separator G3. Check that the applied Applied load is at node 6, $F_Y = 100$ and use SUBMIT G3.

1. Prove reactions equilibrate the applied load.
2. Print M, V, T and draw the bending moment and torque diagrams. Prove that nodes 2 and 5 are in equilibrium under the action of the member node forces.
3. Use the plot command PLTGRD A B (M or R) A=40,30,1.0 N=2 or 4 to view the bending moment diagram and the deflected shape respectively.