ADDED MASS FOR A SINGLE-DEGREE-OF-FREEDOM SYSTEM SUBMERGED IN A FLUID Revision B

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The apparent mass of a body in air differs from that of a mass submerged in a fluid. As a simplification, this effect is due to the buoyancy force acting on the body.

The effective mass m_e of a submerged body is

$$m_e = m + m_a \tag{1}$$

where

m is the physical mass

m_a is the added mass

As a "first approximation," the added mass is equal to the volume of the body multiplied by the density of the fluid. In other words, it is the water mass displaced by the body. This is Archimedes principle.

In reality, the added mass also depends on the body's geometry which affects the acceleration of the fluid particles over the surface of the body. The added mass is the mass of the fluid that must be accelerated in order to allow the acceleration of the body.

Furthermore, the added mass value may depend on the Reynolds number and other variables.

As expected, the fluid also adds damping.

Added mass formulas for sample geometries are given in Appendix A.

Consider the single-degree-of-freedom system in Figure 1. Assume that it is submerged in a fluid.

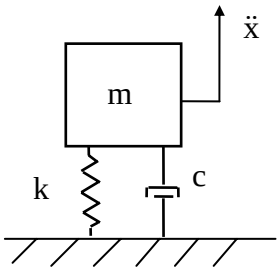


Figure 1.

m	is the mass
c	is the viscous damping coefficient
k	is the stiffness
x	is the absolute displacement of the mass

The added mass opposed the vibration. The equation of motion for "free" vibration is thus

$$m\ddot{x} + c\dot{x} + kx = -m_a\ddot{x} \tag{2}$$

$$\left(m + m_a\right)\ddot{x} + c\dot{x} + kx = 0 \tag{3}$$

$$m_e\ddot{x} + c\dot{x} + kx = 0 \tag{4}$$

The natural frequency fn of the system is

$$fn = \frac{1}{2\pi}\sqrt{\frac{k}{m_e}} \tag{5}$$

References

1. Alexandra Teche, "Object Impact on the Free Surface and Added Mass Effect," MIT, Course Notes, 2004.

2. R. Blevins, Formulas for Natural Frequency and Mode Shapes, R. Krieger, Malabar, Florida, 1979.

APPENDIX A

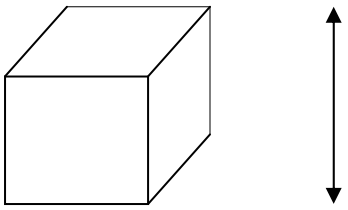
Added Mass Values for Submerged Bodies

The follow formulas are taken from Reference 2.

ρ is the mass of the fluid in the following examples.

The direction of relative flow is indicated by the double-arrow.

Cube

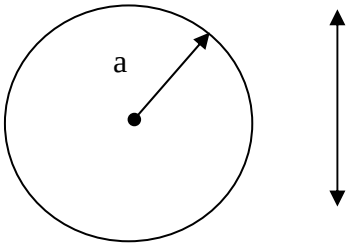


The length of each edge is a. The added mass is

$$m_a = \alpha \rho a^3 \tag{A-1a}$$

$$\alpha \approx 0.7 \tag{A-1b}$$

Sphere



$$m_a = \frac{2}{3}\pi\rho a^3 \qquad \text{(A-2)}$$