SHOCK RESPONSE SPECTRUM SYNTHESIS VIA DAMPED SINUSOIDS
Revision B

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Introduction

Mechanical shock can cause electronic components to fail. Crystal oscillators may shatter, for example. Components such as DC-to-DC converters can detach from circuit boards. Housings and other mechanical parts may develop fatigue cracks, even those made from metal.

Furthermore, mechanical shock can cause temporary malfunctions in addition to hard failures. Mechanical relays can experience chatter, for example.

Design and test specifications are thus needed for base excitation shock. The specification is typically in the form of a shock response spectrum (SRS). The preferred shock testing method is on a shaker table.[1] But pyrotechnic shock levels usually require a mechanical impact or detonation cord method to achieve specifications with high acceleration amplitudes at natural frequencies up to 10 KHz.

Consider the case where a component must survive a pyrotechnic shock pulse, such as a stage separation event in a rocket vehicle. The shock specification is given as an SRS. A finite element analysis is to be performed on the component prior to testing. The analysis is needed to determine the component's relative displacement and acceleration response.

Commercial finite element programs typically perform SRS analyses using some approximate method of combing modal responses, as discussed in Reference 2. These methods appear to be adequate, but they cannot account for the phase between the peak responses of the respective modes.

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[1] A wavelet series can be used for shaker shock testing, as described in Reference 1. Shaker table methods have the advantage of repeatability. Note that shaker tables are limited in terms of displacement, velocity, acceleration, force, voltage, and current, however.

An alternative method is to perform a modal transient analysis using a sample time history waveform that satisfies the specified SRS.[2] This method outputs the response at key nodes in the time domain.[3]

The base input waveform for the modal transient analysis can be synthesized using a series of damped sinusoids. It can also be tailored to plausibly resemble an actual pyrotechnic shock pulse.

Damped Sinusoids

The equation for an individual damped sinusoid is:

$$W_n(t) = \begin{cases} 0, & \text{for } t < t_{dn} \\ A_n \exp\left[-\xi_n \omega_n \left(t - t_{dn}\right)\right] \sin\left[\omega_n \left(t - t_{dn}\right)\right], & \text{for } t \geq t_{dn} \end{cases} \tag{1}$$

where

$W_n(t)$	is the acceleration of damped sinusoid m at time t
A_n	is the damped sinusoid acceleration amplitude, which can be positive or negative
ξ_n	is the damped sinusoid damping ratio
ω_n	is the natural frequency
t_{dn}	is the damped sinusoid time delay

Note that the waveform is lightly-damped such that the damped natural frequency is assumed to be equal to the natural frequency.

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[2] A time history has a unique SRS, but the converse is not true.

[3] The analyst can then study the component response in the time domain, including the excitation and decay. In contrast, the modal combination methods in commercial FEA codes only give peak response values. The modal transient method also has some advantages where dynamic stress and strain are needed.

The total acceleration at any time t for a set of n damped sinusoid is

$$\ddot{x}(t) = \sum_{n=1}^{N} W_n(t) \qquad (2)$$

Selection of the proper damping parameters to reasonably satisfy a given shock response spectrum is a trial-and-error process.

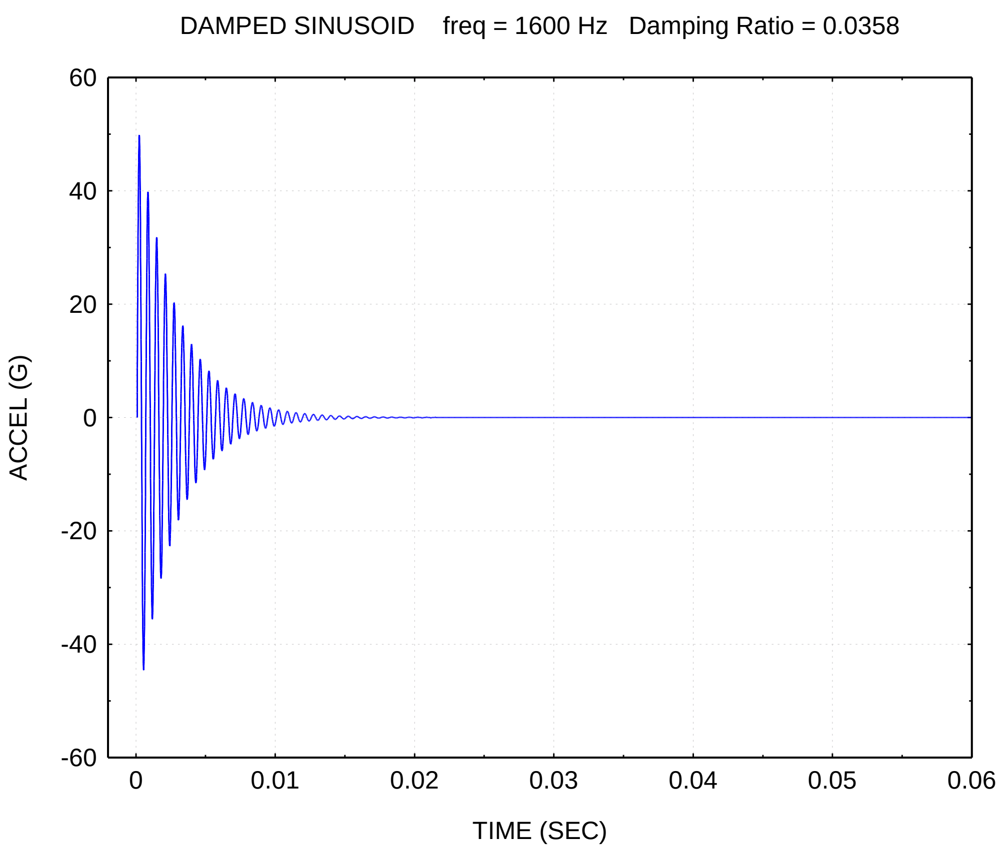

Figure 1.

A time history of sample damped sinusoid is shown in Figure 1. This waveform is used in the following example.

Example

Consider the specification in Table 1. Assume that it represents a pyrotechnic event.

Table 1. SRS Q=10	
Natural Frequency (Hz)	Peak Acceleration (G)
100	100
2000	2000
10000	4000

Notes:

1. Require that both the positive and negative spectral curves meet the specification.
2. Assume tolerance bands of ± 3 dB.
3. Allow a 0.060 second duration.

Synthesize an acceleration time history which satisfies the specification via damped sinusoids.

A synthesis can be performed using the approach in Table 2.

Table 2. Synthesis Steps	
Step	Description
1	Generate random values for the following for each damped sinusoid: amplitude, damping ratio and delay. The natural frequencies are taken in one-twelfth octave steps.
2	Synthesize an acceleration time history from the randomly-generated parameters.
3	Calculate the shock response spectrum of the synthesis.
4	Compare the shock response spectrum of the synthesis to the specification. Form a scale factor for each frequency.
5	Scale the damped sinusoid amplitudes.
6	Generate a revised acceleration time history.
7	Repeat steps 3 through 6 as the inner loop until the SRS error diverges.
8	Repeat steps 1 through 7 as the outer loop until an iteration limit is reached.
9	Choose the waveform which meets the specified SRS with the least error.

The method in Table 2 is a rough outline. Each of the steps can be fine-tuned. Convergence algorithms can also be implemented.

This synthesis method produced the time history shown in Figure 2. The corresponding shock response spectrum is shown in Figure 3. The spectral curves satisfy the tolerance bands.

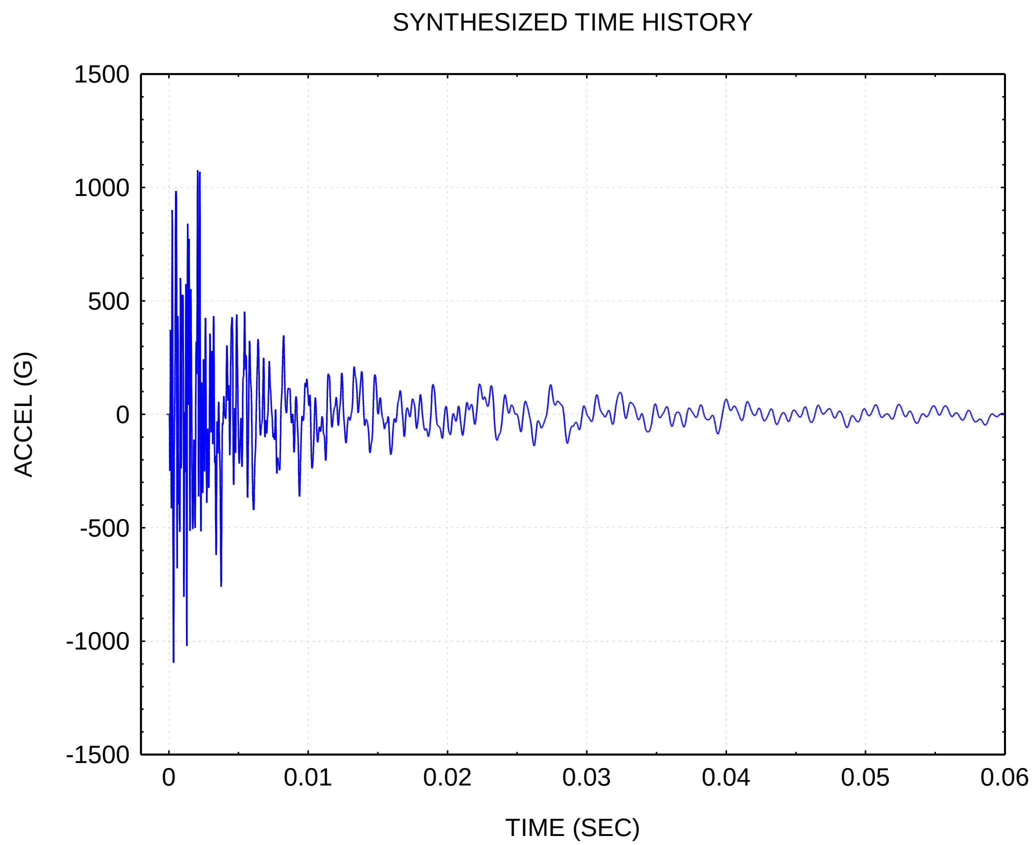

Figure 2.

The synthesized time history resembles a plausible pyrotechnic shock pulse.

It was performed using program damped_sine_syn.cpp version 3.2.

The synthesis consists of 82 damped sinusoids. One of the components is the 1600 Hz waveform from Figure 1.

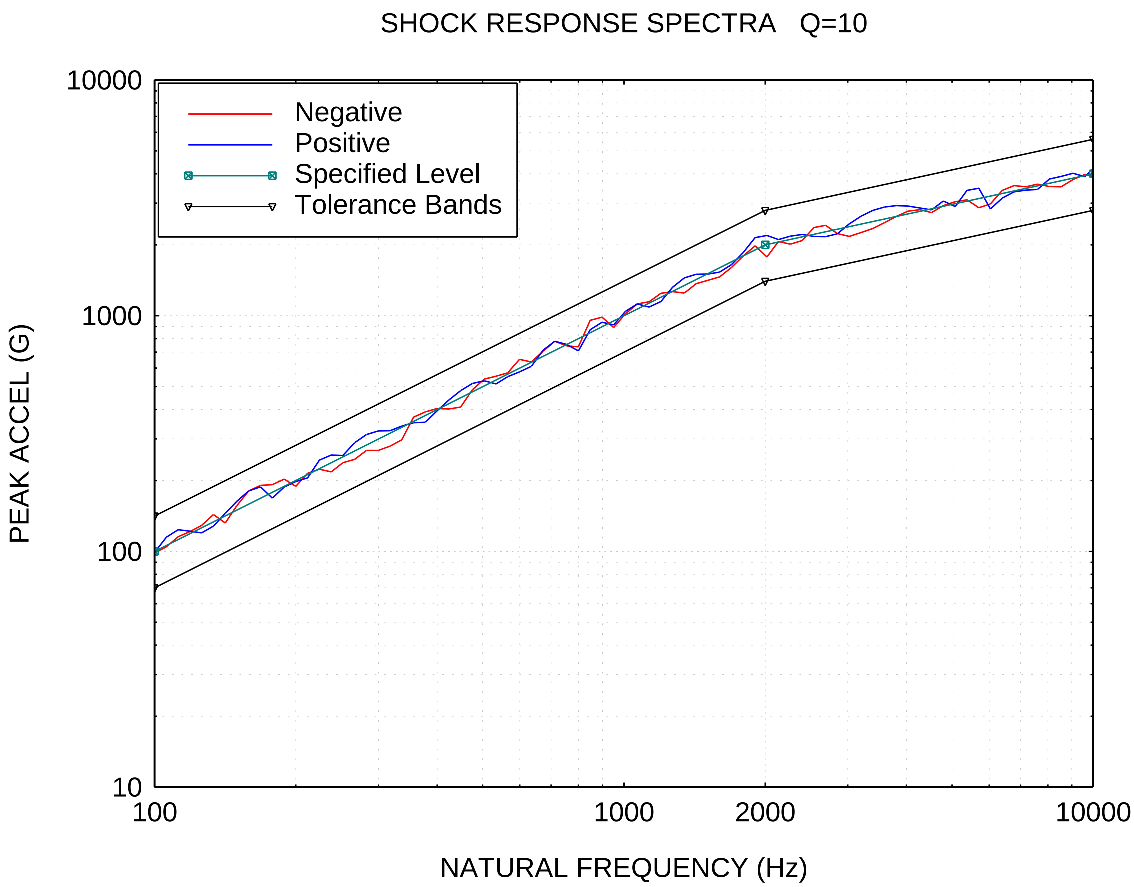

Figure 3.

The positive and negative curves represent the synthesis.

The tolerance bands are $\pm$ 3 dB.

Again, the damped sinusoid method is intended for modal transient analysis, particularly for pyrotechnic shock simulation.

Shaker Shock Testing

The damped sinusoid time history is synthesized in terms of acceleration. The corresponding velocity time history would have a non-zero net velocity change and a non-zero net displacement change.

A compensation pulse would be required to bring the velocity and displacement values to zero at the end of a shaker shock test. Damped sinusoids are rather unsuitable for shaker shock testing for this and other reasons.

Instead, shaker shock testing should be performed using a wavelet series, as discussed in Reference 1.

Waveform Reconstruction

The damped sinusoid series in Figure 2 can be represented in terms of wavelets per the method in Reference 4. The resulting acceleration pulse will have corresponding velocity and displacement pulses which each have net amplitude of zero.

This is shown in Appendix A.

References

1. T. Irvine, Shock Response Spectrum Synthesis via Wavelets, Vibrationdata, 2000.
2. T. Irvine, Shock Response of Multi-degree-of-freedom Systems, Vibrationdata, 2008.
3. T. Irvine, Shock Response Spectrum Analysis via the Finite Element Method, Vibrationdata, Rev B, 2009.
4. R.C. Ferebee, T. Irvine, et al., An Alternative Method of Specifying Shock Test Criteria, NASA/TM-2008-215253, 2008.

APPENDIX A

Acceleration

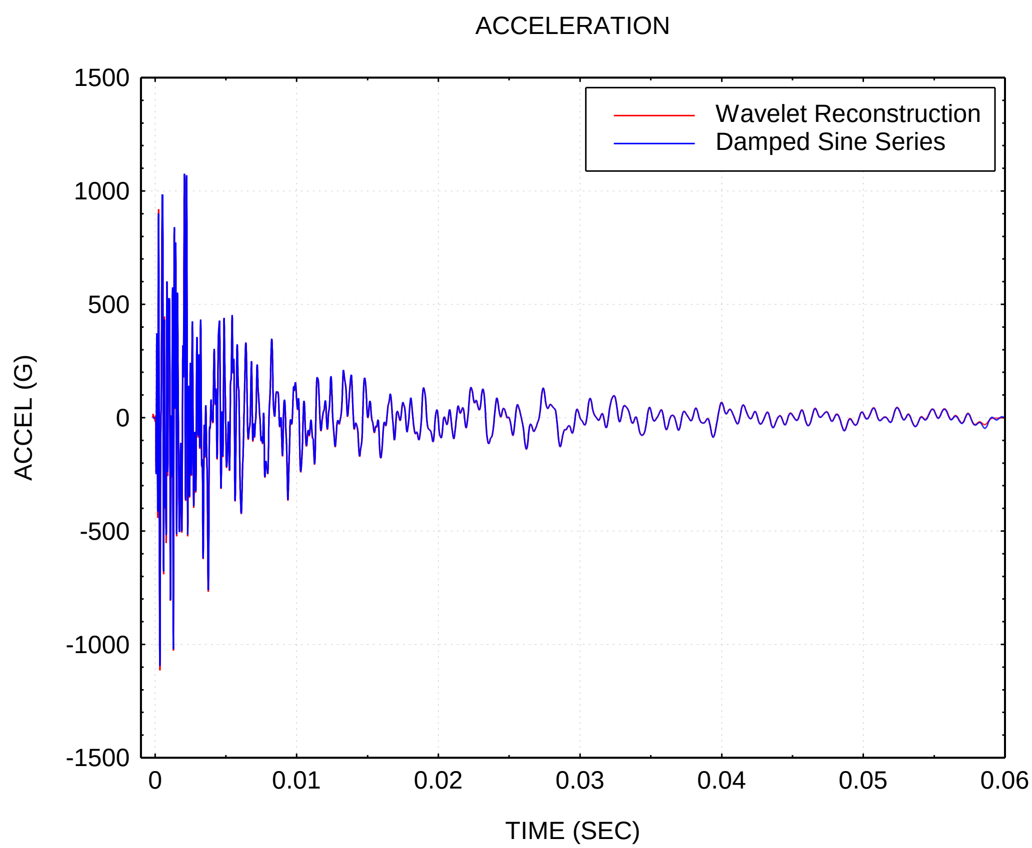

Figure A-1.

The Reconstruction consists of 500 wavelets.

It was performed using program damped_sine_syn.cpp version 3.2 with the reconstruction option.

Velocity

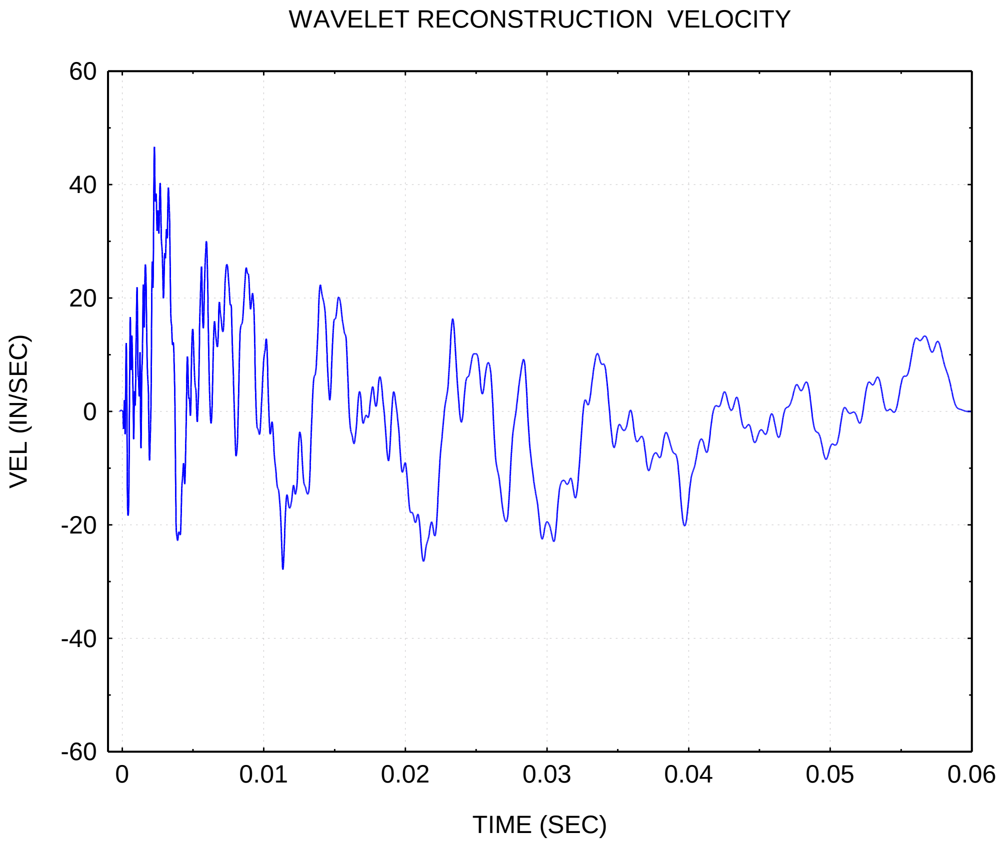

Figure A-2.

Displacement

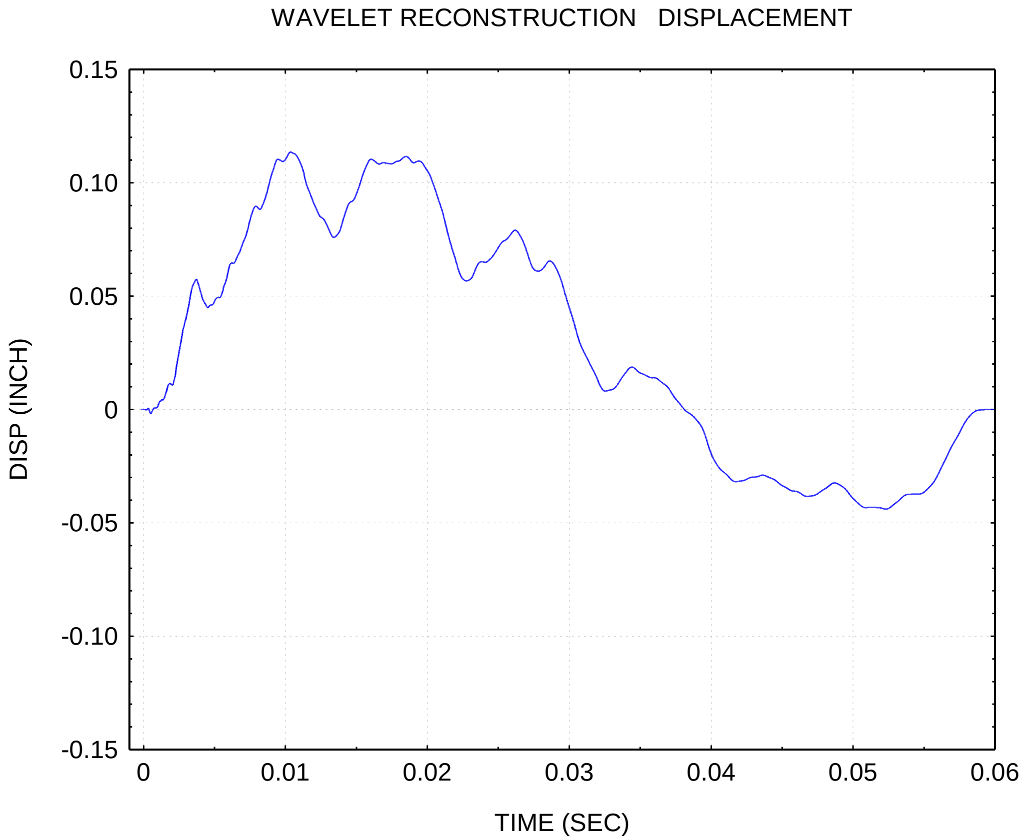

Figure A-3.

SRS Comparison

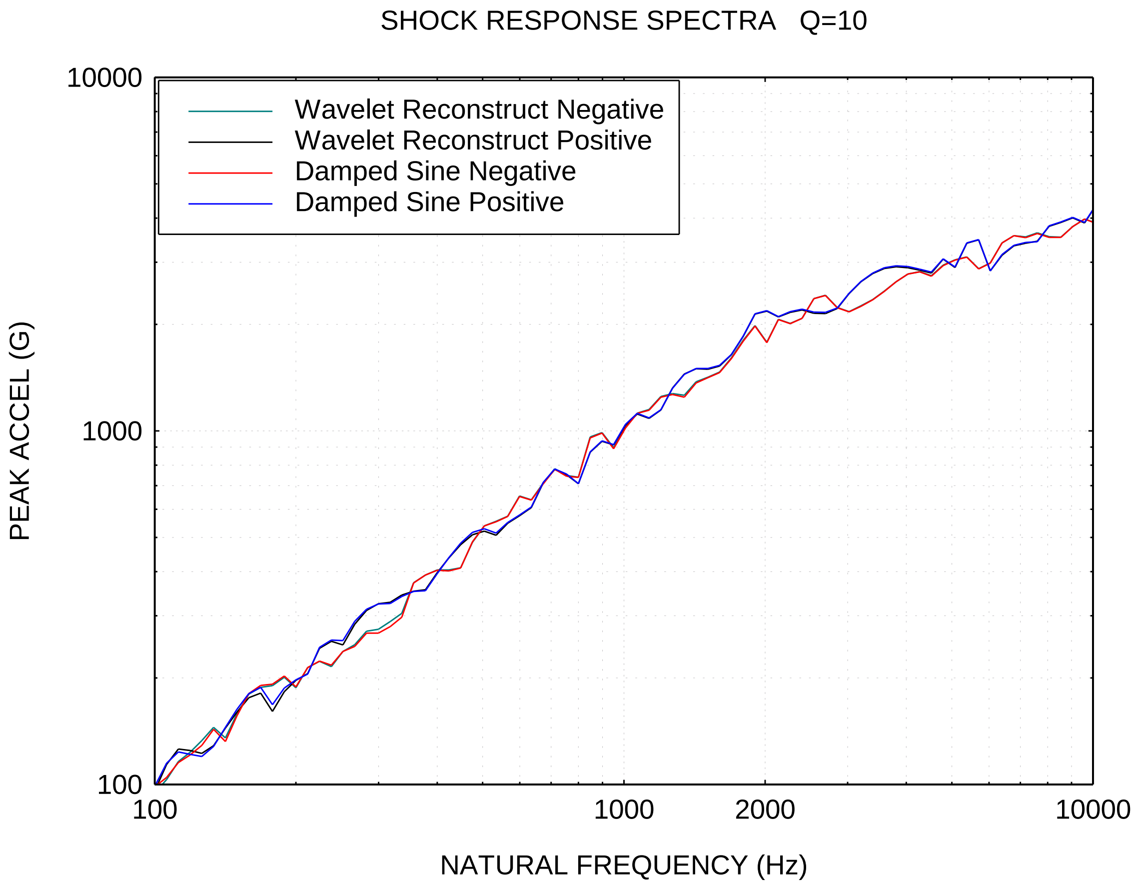

Figure A-4.

The agreement between the Damped Sine and Wavelet Reconstruction is very good for each polarity.